ONE CONTEXT, DIFFERENT PROFILES: CASCADE APPLIED PROBLEMS IN UKRAINIAN SENIOR SECONDARY MATHEMATICS*

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ABSTRACT: *The reform of Ukrainian senior secondary school introduces profile education, while basic-level mathematics is taught under a single programme to students of different profiles. The paper shows how cascade applied problems – context problems with a cascade of interrelated requirements, interdisciplinary connections and a debatable question – can respond to this challenge. We complement the common basic part of a cascade problem with profile continuations and illustrate the approach with the problem "Caffeine and sleep" for Grade 11 (exponential and logarithmic functions). A first trial with pre-service mathematics teachers revealed difficulties in recognising an exponential dependence and in solving the inverse problem; these findings led to revisions of the problem.*

KEYWORDS: *cascade applied problems, context problems, mathematical modelling, profile differentiation, senior secondary school, exponential function*

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1 Introduction

Ukraine is implementing the New Ukrainian School reform [1]. After primary and basic secondary education, the reform now reaches senior (profile) secondary school, grades 10–12, regulated by the State Standard of Profile Secondary Education [2]. Students will choose a profile within one of three clusters – language and literature, social sciences and humanities, and STEM – whereas basic-level mathematics will be taught under a single programme to students of many different profiles [3]. This creates a practical challenge: one mathematics lesson has to be meaningful both for a future engineer and for a future philologist.

Context problems – problems whose situation comes from outside mathematics [10] – are a widely accepted means of meeting this challenge. The PISA framework describes mathematical literacy as the capacity to formulate, employ and interpret mathematics in a variety of real-world contexts [4], and the Ukrainian model curriculum explicitly requires students to model real processes, for instance by means of exponential and logarithmic functions [3]. Empirical studies show that the closeness of a task context to students' experience affects solution success [5], and that learners used to algorithmic problems with a single answer struggle with open applied problems that require their own assumptions and interpretation [6]. At the same time, a real-world context alone does not make a task suitable for modelling. Sitte et al. [7] analysed five widely used Austrian Grade 5 mathematics textbooks: of 443 tasks assigned to cross-curricular themes, 133 were labelled as modelling and problem-solving tasks; real-world contexts were mostly authentic, but tasks combining substantial openness and authenticity were rare, and open goal states were the rarest. Our preliminary observations of Ukrainian mathematics textbooks for grades 5–6 point in the same direction; a systematic analysis is in progress.

In our previous work [8], we introduced cascade applied problems. One of their features is differentiation by the level of students' preparation. The profile school calls for another kind of

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differentiation – by profile. In this paper, we extend the construct in this direction and address the following question: *How can one authentic context be developed into a cascade applied problem that engages students of different profiles in mathematical modelling, and what difficulties do learners encounter when solving it?*

2 Cascade applied problems

Problems connected with a non-mathematical context are named differently in the literature: word problems [9], context problems [10], applications or applied problems [11], modelling tasks [12]. Ukrainian literature uses the terms *applied problems*, *problems with practical content* and, more recently, *competence-oriented tasks*. These terms describe different aspects of a task: the presence of a story, the source of the context, or the activity required. We use *context problems* as an umbrella term. Every applied problem is a context problem, but not vice versa: the context of a problem may be fictional or artificial (for example, two taps filling a pool), whereas an applied problem takes its situation from real life, other sciences or professional practice [11]. In Ukrainian literature, Tarasenkova [16] distinguishes purely mathematical tasks (M-tasks), competence-oriented tasks (KO-tasks) and competence tasks (K-tasks), noting that not every applied problem is a K-task. KO-tasks resemble K-tasks in their plot but are mathematical tasks in structure: translating them into mathematical form uses up all the given data, which is not always the case for K-tasks; she proposes the sequence M → KO → K. Following [7, 13, 14], we regard authenticity as concerning the context, the question and the data of a task, and openness as concerning its initial, intermediate and goal states [15]; Tarasenkova's criterion is close to the openness of the initial state.

According to [8], a cascade applied problem: (1) describes a real or realistic situation that is sufficiently authentic and practically significant for students; (2) contains a cascade of interrelated requirements of increasing complexity, from reproductive to research ones, each building on the result or context of the previous one; (3) necessarily involves interdisciplinary connections; (4) includes a debatable question that has no single mathematically correct answer; (5) is purposefully designed for particular key competencies; (6) provides differentiation, since students with different levels of preparation can complete different numbers of requirements. For the profile school, we complement the common basic part of a cascade problem with **profile continuations**: tasks that develop the same context by means of mathematics and content relevant to particular profiles or clusters. Within one context, a cascade problem thus reproduces the sequence M → KO → K [16]: from reproductive requirements to research ones and a debatable question.

3 Example: "Caffeine and sleep"

The problem is intended for Grade 11 (basic level) at the end of the topic "Exponential and logarithmic functions" of the model curriculum [3]. **Situation.** Maxim is 17 years old and weighs 60 kg. He goes to bed at 11 p.m. but often cannot fall asleep until 1 a.m. In the morning he feels exhausted, so he drinks a cup of coffee at 8:00, another one at 11:00, and a 0.5 l energy drink at 16:00. Maxim believes he sleeps badly because of his phone; his friend says that caffeine is to blame. *Who is right?* **Data.** A cup of coffee contains about 80 mg of caffeine; about an hour after intake the caffeine is fully absorbed, and then the body gradually eliminates it. Table 1 shows the amount of caffeine in the body after one cup of coffee; the data are conditional, constructed from typical pharmacokinetic values for a healthy person (peak after about 1 h, half-life about 5 h) [17]. According to the European Food Safety Authority, up to 3 mg of caffeine per kilogram of

body weight per day is safe for adolescents, and about 1.4 mg/kg consumed before bedtime may prolong sleep latency and shorten sleep in some adolescents [18].

Table 1

Caffeine in the body after one cup of coffee (conditional data)

t, h	1	2	3	4	5	6	7	8	9	10	11
m, mg	75	65	57	49	43	38	33	28	25	22	19

Cascade of requirements. The basic part consists of seven requirements (Table 2). The problem starts with an open initial state: students plot the data and put forward their own hypothesis about the type of dependence. Testing the hypothesis leads to the constant half-life and to the model

(1) $$m(t) = 75 \cdot (1/2)^{(t-1)/5}, \quad t \geq 1.$$

Students then discuss why $m(0) \approx 86$ mg has no real meaning and for which t the model can be used without further assumptions. Applying model (1) to the three intakes gives about 92 mg of caffeine at 23:00, which exceeds the reference value $1.4 \cdot 60 = 84$ mg. Solving the inverse problem, students obtain for themselves that the energy drink should be consumed no later than about 15:00 – a rule that coincides with the common medical advice. The debatable question (task 7) then shows that the rule is not universal: with a half-life of 3 h the latest time shifts to about 19:00, and with 10 h there is no solution at all.

Table 2

Basic part of the problem "Caffeine and sleep"

Requirement	**Mathematical content**	**Openness / authenticity**
1. Hypothesis: plot the data, suggest the type of dependence	Graphs of elementary functions	Open initial state
2. Testing: which quantity must be constant? Half-life	Differences and ratios of values	Open intermediate state
3. Formula, meaning of $m(0)$, domain of the model	Exponential function $m = C \cdot a^t$	Validation of the model
4. Daily dose: find the caffeine content of a real drink	Proportions, percentages	Authentic data found by students
5. Caffeine at bedtime vs reference value	Sum of exponential terms	Interpretation
6. Latest time for the energy drink	Exponential inequality, logarithm	Inverse problem
7. One rule for everybody? Half-life 3 h and 10 h	Parameter variation	Open goal state (debatable question)

The final question returns to the initial dispute: the calculations show that caffeine may contribute to Maxim's sleep problems, but a model of one factor cannot prove that it is *the* cause. **Profile continuations** (Table 3) develop the same context for different clusters; they can be used as homework or project work.

Table 3

Profile continuations of the problem

Cluster / profile	**Content of the continuation**
STEM: "mathematics and physics", "mathematics, informatics, technologies" (advanced level)	Model $m(t) = 75 \cdot e^{-k(t-1)}$; derivative $m' = -k \cdot m$ and its meaning; why a linear model fails; the absorption phase not described by the model
STEM: "chemistry and biology"	Constant fraction eliminated per hour → geometric progression; reliable sources on factors that speed up or slow down elimination; share of caffeine left at 23:00 for different half-lives

Social sciences and humanities; language and literature	Anonymous class survey on caffeinated drinks; mean, median, mode; checking the claim "all senior students drink coffee"; limits of generalising from a small non-random sample

The basic part is designed for mathematical competence and competence in natural sciences (building and validating a model of a natural process), information and digital competence (finding and evaluating data), and civic competence (arguing and changing one's mind on the basis of evidence); the context also supports the value of a healthy lifestyle. The continuations show that the same competence – for instance, evaluating the reliability of information – is developed on different material: through the inadequacy of a linear model in STEM, the choice of reliable sources in the chemistry and biology profile, and testing a popular claim against survey data in the humanities.

4 First trial

The basic part was first discussed with two fourth-year pre-service mathematics teachers at Zhytomyr Ivan Franko State University in September 2026: one student worked at the blackboard, the other at the desk, and the teacher moderated the discussion. The trial was not intended to evaluate the problem but to reveal where it needs support. Four observations emerged. First, the open start worked as intended: the students suggested a straight line and a hyperbola as hypotheses, and the line visibly failed to fit the data. Second, the students noticed nearly constant ratios of neighbouring values but did not conclude that the dependence was exponential; moreover, the original task 2 checked only the linear hypothesis, leaving the hyperbola hypothesis without a test. Third, both students assumed that the graph of an exponential function must pass through the point (0; 1): they knew $y = a^x$ but not $y = C \cdot a^x$. This is a case of a narrow concept image as opposed to the concept definition [19], and it concerns future teachers, not only school students. Fourth, task 6, the first inverse problem with time as the unknown, caused a clear pause.

Accordingly, the problem was revised (Table 4). Following research on instructional prompts for open modelling problems [15], the hints are given only on request, so that the openness of the problem is preserved for stronger students.

Table 4

Difficulties observed and revisions of the problem

Difficulty	Revision
Constant ratios noticed but not linked to exponential dependence; no test for the hyperbola hypothesis	Task 2: "Which quantity must be constant if your hypothesis is correct? Check it in the table." Hint card: constant differences – linear function; constant product $t \cdot m$ – inverse proportionality; constant ratio at equal steps – ?
Belief that an exponential graph must pass through (0; 1)	Hint card for task 3: "Through which point does the graph of $y = C \cdot a^x$ pass? Must it pass through (0; 1)?"
Pause at the inverse problem (task 6)	Hint card: let the drink be taken at x o'clock – how many hours remain until 23:00? How many times more caffeine does the drink contain than a cup of coffee?

The trial has obvious limitations: only two participants, pre-service teachers rather than school students, and active support of the teacher. Its results are therefore used to revise the problem, not to evaluate its effectiveness.

5 Conclusions

The example shows how one authentic context can be developed into a cascade applied problem for students of different profiles: a common basic part leads all students from their own hypothesis to an exponential model, a self-derived rule and a debatable question, while profile continuations develop the same context by means of mathematics and content relevant to different clusters. The first trial suggests that the main difficulties lie not in calculations but in recognising the type of dependence and in the inverse problem, and that hint cards given on request can support students without closing the problem. Further work includes a trial with Grade 11 students of a physics and mathematics profile, the development of further cascade problems (for instance, on the stability of plant stems), and preparing pre-service teachers to design such problems, including with the help of artificial intelligence tools.

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