MODELLING INCOME MOBILITY AND FINANCIAL RESILIENCE THROUGH MARKOV CHAINS: SURVEY EVIDENCE FROM BULGARIA*

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ABSTRACT: *Income should not only be interpreted as a current monetary indicator but rather that it is a dynamic socio-economic position that may change over time. A Markov chain model is applied to survey data to explore income mobility in Bulgaria. Two transition models were estimated. The investigation is extended through indicators of financial resilience including saving income stability and the ability to cover unexpected expenses. The findings indicate that there is income mobility and it is not mainly upward. The middle income category is the most stable category. According to the results, mobility can carry economic consequences; in particular, upward mobility is associated with higher financial buffers while downward mobility is linked to greater vulnerability. The paper shows that Markov chains are clear and useful for modelling income transitions, along with measures of financial resilience.*

KEYWORDS: *income mobility; Markov chains; transition matrix; financial resilience; intergenerational mobility; intragenerational mobility; Bulgaria.*

DOI: https://doi.org/10.46687/PYOM2670

1 Introduction

One of the main measures that can be used to assess the extent to which a society is economically open is income mobility. Not all contexts have equally serious problems with low income jobs. In the first case, the term applies when people have a realistic chance of progressing upward, while in the second, it applies when income positions are inherited across generations or fixed over the life cycle. Consequently, it is improper to restrict the study of income to means, wage levels, and income groups at one point in time. It should study the likelihood of switching from one position to another.

The concept of income mobility is widely discussed in the literature as a way of measuring changes in the economic status of individuals or households over time [1]. The broader inquiry of social mobility is closely related here. To what extent does family background shape future economic outcomes? The OECD uses the metaphor of a "broken social elevator" to describe situations in which social origin continues to strongly determine individual opportunities [2]. In this sense, income mobility is not simply a statistical concept, but in many regards, a measure of economic fairness, opportunity and social resilience.

The paper focuses on modeling income mobility as a stochastic process with a finite number of states. The Markov chain method is useful when we want to estimate the probability of moving from one state to another. In this study, income status is represented through three states: low, middle and high-income status. The study evaluates two models. The first measure of intergenerational mobility compares a respondent's family income background when they were children to their current income position. The second one compares the respondent's income position five years ago to the current income position to measure intragenerational mobility.

The paper's empirical contribution is that income mobility cannot be analysed in isolation. It is read alongside financial durability. Upward mobility does not equal economic security. Having a higher income does not guarantee the ability to go through financial shocks. Consequently, the study investigates the association between mobility type and a range of financial resilience outcomes such as savings and the ability to cover unexpected expenses.

The paper aims at showing how Markov chain modelling can help describe income mobility, and how the resulting transition structure can be interpreted in relation to financial resilience. The income mobility and financial resilience of respondents in Bulgaria are the object of the study, and the transition between different income positions is the subject. The research tasks are to construct the intergenerational and intragenerational transition matrices, to estimate the mobility indicators and the stationary distributions, and to interpret the results through cross-sectional measures of financial resilience.

2 Survey data and methodological framework

Empirical analysis from the survey conducted to examine income position, income mobility and financial resilience has been analysed in this chapter. The survey observed questions about the financial position today, self-assessed position relative to the average in the country, family income background during childhood, changes in financial position over the last five years, labour market status, sources of income, savings, ability to meet unforeseen expenses, education, region of residence and basic demographic characteristics. In total 240 responses were collected.

The design of the survey permits that income position can be studied not only as a current status but also as a transition. Therefore, the descriptive survey analysis and Markov chain modelling are combined in the empirical part. The number of valid observations of the models differ because not all respondents answered the questions used to construct the transition matrices. The intergenerational Markov model is formed from 228 valid observations while the intragenerational Markov model is analysed from 235 valid observations.

For the purposes of the Markov analysis, income position is classified into three states:

S_1 - low income position;

S_2 - middle income position;

S_3 - high income position.

The intergenerational model originates in a respondent's family income background in childhood and ends in the present income position of the respondent. In the intra-generational model, the initial state is indicated by the self-assessed income position five years previously and the final state is the current income position. This structure enables mobility comparisons across two horizons: between generations and along the respondent's own recent economic trajectory.

The movement between these states is modelled through a transition probability matrix (1):

$$P = (p_{ij}), \tag{1}$$

Where p_{ij} denotes the probability of moving from state i to state j. In the context of the present study, this means:

$$p_{ij} = P(X_{t+1} = j \mid X_t = i).$$

The empirical transition probabilities are estimated through the relative frequencies of observed transitions (2):

$$\hat{p}_{ij} = \frac{n_{ij}}{\sum_{j=1}^{k} n_{ij}}, \tag{2}$$

where n_{ij} is the number of respondents observed in transition from state i to state j. This approach is consistent with the standard estimation of transition probabilities in finite Markov chains [3], [4].

In order to assess the uncertainty of the estimated transition probabilities, approximate 95% confidence intervals are calculated with the usage of the normal approximation for a

proportion. The interval is defined as: $\hat{p}ij \pm 1.96 \cdot \sqrt{\frac{\hat{p}ij(1-\hat{p}ij)}{n_i}}$, where $\hat{p}_{ij}$ is the estimated transition probability and n_i is the number of respondents in the initial income state.

The stationary distribution of the chain is obtained from $\pi P = \pi$ with the condition $\pi_1 + \pi_2 + \cdots + \pi_i = 1$. The stationary vector π shows the long-run distribution of income positions if the observed transition probabilities remain unchanged. This should not be interpreted as a deterministic forecast but as a scenario which describes the internal tendency of the transfer system.

Three mobility indicators and transition probabilities are used. The proportion of respondents who stay in the same income category is the first indicator. The second is the overall mobility rate, which is the share of respondents moving to a different income position. The third is the Shorrocks mobility index, which summarises mobility through the trace of the transition matrix (3); [5]:

$$M_S = \frac{k - tr(P)}{k-1}, \quad (3)$$

where *k* is the number of states and *tr(P)* is the trace of the transition matrix. A higher value of the index indicates higher mobility.

The cross-sectional association measures also examine income mobility and financial resilience relationship. The Pearson chi-square statistic is employed in the case of categorical variables, and Cramer's V summarises the strength of association.

3 Intergenerational income mobility

The first Markov model examines the transition from family income background in childhood to the respondent's current income position. The transition matrix is presented in Table 1.

Table 1

Intergenerational transition matrix, %

Family income background	Low current position	Middle current position	High current position
Low family background	23.91	60.87	15.22
Middle family background	25.35	59.86	14.79
High family background	30.00	50.00	20.00

Regardless of background, the results show the dominance of middle income position. As many as 60.87% of respondents from low-income families currently belong to the middle income group while 15.22% move to a high income position. Among respondents from middle-income families, 59.86% remain around the middle position and 14.79% report a high current position. The group from the upper-class family shows concentration in the middle position, but family upper-class groups also have the highest probability of staying or moving to the high position of 20.00%.

We can interpret that family income background matters but it does not fully determinates current income position. There is visible mobility, especially from low and middle origin towards the middle income position. At the same time, the high-income position does not takes to strong expansion, even for respondents from a high-income family background.

The approximate 95% confidence intervals are also in support to this idea. For the transition to the middle-income position, the intervals are 46.77% - 74.97% for respondents from

low-income families, 51.80% - 67.92% for respondents from middle-income families, and 34.50% - 65.50% for respondents from high-income families. This confirms that the middle-income position remains the dominant outcome across different family backgrounds.

The mobility structure is summarised in Table 2.

Table 2

Mobility indicators: intergenerational and intragenerational models

Indicator	Intergenerational mobility	Intragenerational mobility
No change (%)	45.61	63.40
Upward mobility (%)	24.56	13.19
Downward mobility (%)	29.82	23.40
Overall mobility index (%)	54.39	36.60
Shorrocks mobility index	0.9811	0.6266

According to the data for the intergenerational model, 45.61% of responses are for those who remain in the same broad income position. 54.39% evidence of any type of movement. Upward mobility is 24.56%, but downward mobility is 29.82%. This structure is significant: there is mobility, but not mainly upward. The greater share of downward mobility compared to upward mobility suggests that income position is not just a channel of opportunity but also a field of risk.

The stationary distribution of the intergenerational Markov chain is $\pi = (0.2571, 0.5857, 0.1572)$. In other words, the long-run income distribution would settle at 25.71% in the low position, 58.57% in the middle position, and 15.72% in the high position under the observed transition structure. The system, therefore, makes the middle group dominant, while the low-income group remains large, and the high-income group remains quite limited.

4 Intragenerational income mobility over a five-year period

The second Markov model looks at income mobility in the respondent's recent economic background. The respondent's self-assessed income position five years earlier is the initial state while the final state is the respondent's own current income position. This model is important because it captures short-term mobility and tells us whether or not position changed over a limited time period.

Table 3

Intragenerational transition matrix, %

Income position five years ago	Low current position	Middle current position	High current position
Low position five years ago	52.08	45.83	2.08
Middle position five years ago	21.48	72.59	5.93
High position five years ago	9.62	40.38	50.00

The findings in Table 3 reveal that the intragenerational model has greater persistence than the intergenerational model. The mid-income grouping has the largest stability: 72.59% of respondents that were in the middle position five years ago remain in the middle position at the time of the survey. 52.08% of the members of the low-income group also show persistence. Similarly, 45.83 % on the average of the low move up to the middle, which is upward movement

although it is mostly movement towards the middle rather than directly towards the high-income category.

The high income position is not completely stable. Of all respondent who are above average five years back, 50.00% stay in high position, 40.38% move to middle income group. The result is significant since it suggests a higher income position does not always imply a permanent advantage. As a result, income mobility is not only a mechanism for rise but also a process that entails a risk of fall.

The approximate 95% confidence intervals for remaining in the same income position are 37.95% - 66.22% for the low-income group, 65.07% - 80.12% for the middle-income group, and 36.41% - 63.59% for the high-income group. These intervals add a further support to the conclusion that the middle-income category is the most stable position in the short-run mobility model.

The stationary distribution of the intragenerational Markov chain is $\pi = (0.2949, 0.6194, 0.0857)$. This means that, under the observed five-year transition structure, the system tends towards a distribution in which 29.49% of respondents are located in the low income position, 61.94% in the middle position and only 8.57% in the high position. Compared with the intergenerational model, the intragenerational stationary distribution shows an even stronger concentration around the middle income state and a smaller long-run share of the high income category.

A clear conclusion is derived from the comparison of the two Markov models. Intergenerational mobility is characterized by greater dynamics than intragenerational mobility is. Changes in education, occupation, family trajectory and social environment over a longer intergenerational horizon offer more opportunities for movement. However, on a shorter five-year horizon, income positions are more persistent with less variation in movements. Income mobility depends on the time horizon for the analysis that one performs.

5 Cross-sectional evidence: income mobility and financial resilience

Markov models characterize the directional movements between income positions while cross-sectional analysis unclears the economic meaning of these movements. The findings suggest that the current income position and mobility type relate to financial resilience. Individuals with a more favorable current income situation are more likely to report savings, better income stability, and the ability to cover an unexpected expense of 1000 euro. The strongest associations are observed between current income position and savings (Cramer's V = 0.3620), current income position and ability to cover an unexpected expense (Cramer's V = 0.3598), and current income position and income stability (Cramer's V = 0.3310).

Income mobility is also connected with financial resilience. Among respondents with upward intergenerational mobility, 82.14% can cover an unexpected expense of 1000 euro without difficulty, compared with 45.19% among respondents with no change and only 22.06% among downwardly mobile respondents. A similar pattern is observed for savings: 80.36% of upwardly mobile respondents have savings for at least three months, while the share among downwardly mobile respondents is only 36.76%. This shows that upward mobility is associated with stronger financial buffers, whereas downward mobility is linked to higher vulnerability.

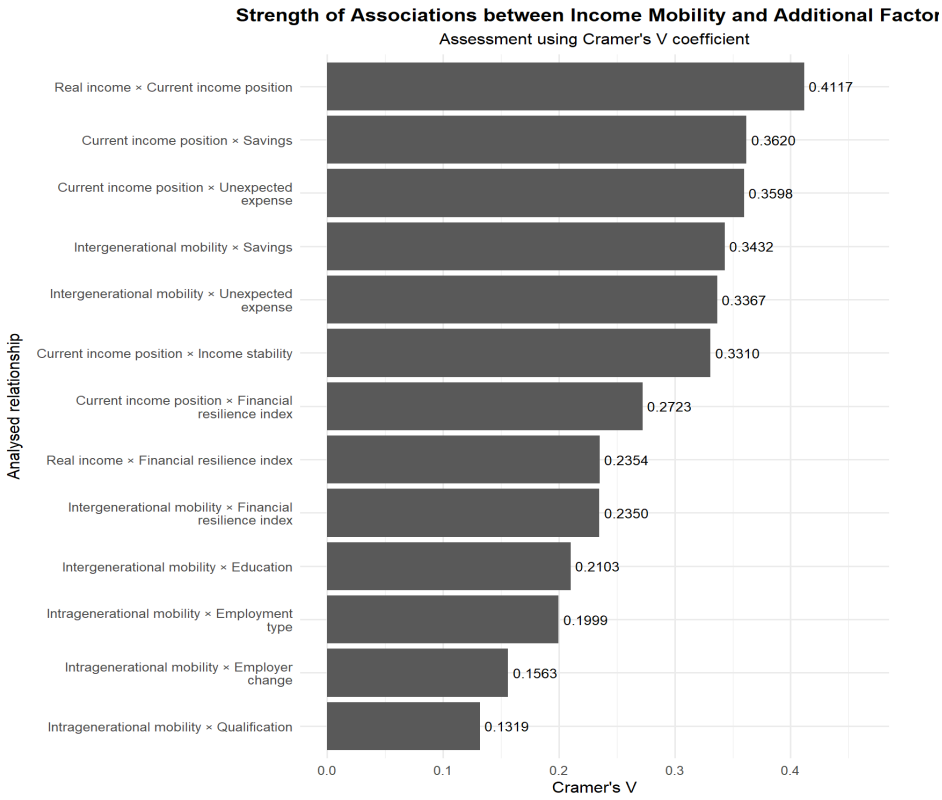


Figure 1 Strength of associations between income mobility and additional factors

Figure 1 summarises the strength of the tested relationships through Cramer's V. As it turns out, financial indicators are the most important associates. In comparison with that, relationships with education, qualification, employer change and type of employment have a weaker association. Consequently, the cross-sectional analysis confirms that an advancement or decline on an income scale involves more than just shifting from the person below to the person above one's category. It is in itself a process that has direct effects on financial security.

6 Mathematical interpretation of financial security

The empirical results can be linked to a simple life-cycle interpretation of financial security. Let *Y(t)* denote income at time *t*, and let *W(t)* denote accumulated financial resources or wealth. Income may be represented as an exponential growth process:

$$Y(t) = Y_0 e^{gt},$$

where Y_0 is initial income and g is the growth rate of income. Accumulated financial resources may be described through the differential equation $\frac{dW(t)}{dt} = rW(t) + sY(t)$, where r is the return on accumulated resources and s is the savings rate from current income. The equation shows that financial security depends on two channels: the return on already accumulated resources and the ability to transform current income into savings.

When $g \neq r$, the solution can be written as $W(T) = \frac{sY_0}{r-g}(e^{rT} - e^{gT})$.

This new model provides a formal interpretation of the empirical data. Being in a higher income position is important; but financial security also relates to capacity to save, stability of incomes and to accumulation of reserves over time. This explains why two people on similar incomes may have very different levels of financial resilience.

Moreover, the model clarifies the economic risk of downward mobility. If an individual's income position changes in a negative direction, such a change would reduce their current income. Further, it may stop the process of saving and reserve accumulation. When financial cushions deplete, even a temporary shock may create longer-term vulnerabilities. That's why income mobility and financial resilience should be analysed jointly.

7 Discussion and conclusion

The findings indicate that income mobility in the studied sample in Bulgaria is limited. Family income background affects current income position of the respondent as per intergenerational model but does not completely predetermine it. Over a 5-year period, the intragenerational model displays greater persistence, especially in the middle income range. The middle income position seems to be the stabilising core of the system in both models.

The finding that mobility is not mainly upward is important. Both Markov models show downward mobility, which should be interpreted as a relevant economic risk. The present study shows that the probability of moving from a high to a lower income position over a five-year period is 50.00%, while 80.00% of respondents from a high-income family background are currently in a lower income category. This reminds us that inherited advantage matters, but it does not fully determine socio-economic status. From a practical socio-economic perspective, income mobility cannot be understood only as a possibility of upward movement, but also a risk of falling to a lower-income level. To reduce this risk policy measures should focus on support for stable employment, financial literacy, savings capacity and emergency financial buffers for vulnerable households. The stationary distributions endorse this conclusion because the system tends to cluster around the mid-income position while the high-income category is rather limited.

The cross-sectional analysis provides a crucial economic interpretation of the results. Upward mobility can increase the individual's financial stability, savings and cushion against shocks. On the contrary, downward mobility is linked to weaker financial buffers and higher vulnerability. Consequently, we cannot think of income mobility merely as movement between income groups; it is also a process that enables individuals to withstand financial shocks.

The research proves that Markov chains are interpretable and useful for modelling income transitions. When transition probabilities are interpreted together with indicators of financial resilience, more analytic value is observed. One limitation of the research is that the data are based on a survey sample and should not be interpreted as fully representative of the entire Bulgarian population. Nonetheless, the outcomes show that income mobility and the subject of financial security can be correlated, and they need to be studied together at any point of time when making an assessment of the socio-economic position of individuals.

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