SYNONYMIC VARIATION UNDER SEMANTIC CONSTRAINT IN MATHEMATICAL DISCOURSE

VANYA A. IVANOVA, ANGEL Y. TODOROV, BOYAN G. ZLATANOV

ABSTRACT: *This paper examines context-dependent alternatives to constructions introduced by* from *in mathematical discourse. Through a qualitative analysis of constructed examples, it distinguishes reference to a source, application of a result, succession, and combination of premises. Expressions such as* using, according to, *and* together with *may support the same conclusion, but they need not preserve the same account of how that conclusion is obtained. We therefore distinguish preservation of mathematical content from preservation of the inferential relation and rhetorical emphasis. The analysis identifies conditions under which reformulation is appropriate and illustrates failures caused by missing hypotheses, altered quantifier scope, or changes in the mode of convergence. A conceptual discussion connects controlled revision with order-based enrichment, fixed points, and alternating revision processes without asserting convergence for unrestricted editing. The resulting guidelines treat lexical variation as subordinate to mathematical precision.*

KEYWORDS: *Mathematical discourse; synonymic variation; semantic constraints; mathematical English; academic writing; functional equivalence.*

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1 Introduction

Mathematical exposition relies on recurrent ways of introducing assumptions, citing results, and deriving conclusions. Research on the linguistic demands of mathematics education and on formulaic language provides a broader context for examining these patterns [11, 13]. Repetition is not necessarily a stylistic defect: a recurrent expression can make the structure of a proof easier to follow. Variation is useful when it clarifies that structure or improves readability, but it should not obscure a logical relationship merely to avoid repeating a word.

Lexical relations such as synonymy are context-sensitive [7, 8, 9]. Expressions that can replace one another in a particular sentence may differ elsewhere in meaning, syntax, or register. This issue is relevant to English for Specific Purposes (ESP), including courses for students of mathematics and informatics. Work on academic vocabulary, language in undergraduate mathematics, and metadiscourse offers the broader educational and rhetorical background [10, 1, 3, 12]. Here we focus on a narrower question: under what conditions can an alternative to a construction with *from* preserve the intended mathematical relation?

Ordinary English resources acquire specialized uses in mathematical exposition. For example, *let* can introduce an object: "Let x be a real number." The instruction "Assume that f is continuous" establishes a hypothesis about a function f; it need not express uncertainty about that hypothesis. These are specialized uses of familiar constructions, rather than evidence that their ordinary meanings have disappeared.

The preposition *from* frequently introduces the basis of a deduction: "From the definition, it follows that ..." or "From the preceding inequality, we obtain ...". Possible reformulations include constructions with *by*, *using*, *in view of*, or *as a consequence of*. Others, such as *according to*, *following*, and *together with*, require particular contexts or more substantial changes to the sentence.

We use *transition expression* as a descriptive umbrella term for these forms. They belong to different grammatical categories and are not all discourse markers in a narrow grammatical

sense. The term *synonymic variation* in the title accordingly refers to context-dependent functional alternatives, not to unrestricted lexical synonymy. Our main distinction is between preserving a mathematical conclusion and preserving the relation by which the writer presents that conclusion as justified.

2 Scope and Analytical Procedure

This paper is a qualitative conceptual analysis of constructed examples. The sentences are illustrative, not quotations from a corpus or findings from a reader experiment. We compare selected non-specialist and mathematical contexts; we do not claim that the examples establish usage frequencies, historical change, or a measurable degree of semantic neutralization. The examples were selected to contrast inferential, attributive, instrumental, sequential, and combinatory readings. Judgements of naturalness are the authors' analytical judgements, not independently validated ratings. The contextual functions discussed below are analytical tendencies rather than an exhaustive classification of each expression.

A proposed reformulation is examined against four questions:

1. Does it preserve the mathematical claim, including its hypotheses, quantifiers, and technical terms?

2. Does it preserve the relevant relation, such as inference, attribution, temporal succession, or combination of premises?

3. Is the resulting sentence grammatically complete, with the appropriate complement and a clear subject for any introductory participial phrase?

4. Does its rhetorical emphasis suit the argument and the intended audience?

Two versions may preserve the same claim while differing in the second or fourth respect. We call them *content-preserving alternatives*; we reserve *equivalent reformulations* for equivalence with respect to both the mathematical claim and its justification. Neither classification implies that all stylistic differences disappear. This distinction also prevents a grammatically possible sentence from being treated automatically as an appropriate explanation of a proof step.

3 Semantic Distinctions in Non-Specialist English

Consider "According to the doctor, the treatment is effective." Here *according to* attributes the statement to a source; it does not itself endorse the source or prove the statement. The version "Using the doctor, the treatment is effective" does not preserve that relation and introduces an unclear instrumental reading. Its failure cannot be explained by synonymy alone, because the substitution also changes the grammatical relationship between the opening phrase and the subject.

Similarly, "Using the prescribed treatment, the patient recovered" presents the treatment as a means, whereas "According to the treatment, the patient recovered" would require an unusual interpretation of the treatment as a source of information. The latter is unsuitable for the intended reading, although a special context could change the judgement.

"Following the storm, the roads were closed" places the closure after the storm. The sequence may suggest causation in context, but the expression does not by itself entail it. "Because of the storm, the roads were closed" explicitly presents a cause. Thus a change that leaves the reported events unchanged can still change the asserted relation between them.

"On the basis of the evidence, the court reached a decision" identifies the grounds of a decision. "Via the evidence, the court reached a decision" is less natural for that purpose, though *via* can introduce a route or an intermediary in other contexts. In "The information was transmitted via satellite", the expression identifies a channel. Replacing it by *according to* would change the construction from transmission to attribution.

Finally, "Combining the witness statements with the physical evidence, the investigators reconstructed the event" explicitly identifies two inputs. "According to the witness statements with the physical evidence, the investigators reconstructed the event" does not preserve that structure. This contrast involves both the relation expressed and the complement pattern required by the construction.

These examples distinguish attribution, means, cause, succession, grounds, channel, and combination. They do not imply that each expression has a single meaning across all contexts, or that every unsuccessful substitution is a purely semantic error.

4 Contextual Overlap in Mathematical English

A mathematical context can make several formulations compatible with one argument. Suppose a theorem directly supplies the estimate required in a proof. The writer may say "By the theorem, we obtain the estimate" or "Using the theorem, we obtain the estimate". The first identifies a justification; the second foregrounds application. "According to the theorem, the estimate holds" is appropriate when it reports a conclusion of the theorem. If additional reasoning is required, the writer should make that reasoning visible rather than imply that the desired estimate is stated directly in the cited result.

These alternatives illustrate contextual overlap, not a general disappearance of semantic distinctions. The discussion is compatible with research on disciplinary metadiscourse [3, 4], but the specific contrasts here are supported by the constructed examples and the analysis of their logical structure.

Other replacements need greater care. "Following the theorem, we obtain the estimate" can suggest position in the exposition or continuation after the theorem. If the intended meaning is deduction, "It follows from the theorem that the estimate holds" is clearer. If the writer follows a method, "Following the proof of the theorem, we obtain ..." identifies that method more accurately.

The construction *together with* requires another contribution. "Together with the theorem, the preceding inequality yields the estimate" names two inputs. It is an appropriate reformulation only if both actually contribute to the deduction. Similarly, "Combining the theorem with a lemma, we obtain the estimate" cannot replace a sentence citing the theorem alone without checking whether the lemma is an additional premise.

The phrase *in accordance with* usually presents conformity with a rule, convention, or specification. A conclusion may be compatible with a theorem without having been deduced from it. Likewise, *due to* and *owing to* can have explanatory uses, but they are not uniformly suitable replacements for an inferential construction. *Via* can identify an intermediate method or result, provided that route is clear.

Syntactic control matters as well. "Using the decomposition, we split the integral" assigns the action of using to the authors. "On the basis of the decomposition, we split the integral" expresses the grounds of the step. Both avoid the possible attachment problem in "Based on the decomposition, we split the integral", where the introductory phrase may appear to modify *we*.

Thus, a proof can support several descriptions of the same step while retaining meaningful

distinctions among them. The most useful revision makes the relevant relation explicit; it need not maximize lexical variety.

5 Mathematical Examples

The following examples are self-contained illustrations of the distinctions above. Their mathematical assumptions are stated so that the linguistic comparison does not depend on an incomplete or incorrect mathematical claim.

Example 1 (Reference to a stated result)**.** *Let H be a separable Hilbert space and let $A : H \to H$ be a compact self-adjoint linear operator. According to the spectral theorem, H admits an orthonormal basis consisting of eigenvectors of A.*

Here *according to* reports the content of a theorem, subject to its hypotheses. The basis includes eigenvectors in the kernel when the kernel is nontrivial. The sentence does not treat authority as a substitute for checking applicability. A continuation beginning "Using this basis, we represent ..." would instead emphasize an operation performed with the result.

Example 2 (Application of an inequality)**.** *Let $p, q \in (1, \infty)$ satisfy $1/p + 1/q = 1$. For measurable functions $f \in L^p([0,1])$ and $g \in L^q([0,1])$, using Hölder's inequality, we obtain*

$$\|fg\|_1 \le \|f\|_p \|g\|_q.$$

Here $L^r([0,1])$ is the space of measurable functions with finite rth-power integral, identified up to equality almost everywhere, and $\|h\|_r = (\int_0^1 |h(t)|^r \, dt)^{1/r}$ for $1 \le r < \infty$; the integral is the Lebesgue integral.

The expression *using* foregrounds application. "By Hölder's inequality" would preserve the same justification. Neither version permits omission of the conjugacy condition on the exponents when it has not already been established.

Example 3 (Succession in the exposition)**.** *Let $(x_n)_{n \ge 1}$ be a bounded real sequence, where n is a positive integer. Following this choice of sequence, we define $M = \sup\{x_n : n \ge 1\}$, its least upper bound.*

The phrase *following this choice of sequence* organizes the exposition. The existence of the finite real number M is justified by boundedness and the least-upper-bound property of the real numbers, not by the fact that one sentence follows another.

Example 4 (Dependence on a decomposition)**.** *Let $a < c < b$ be real numbers and let f be Riemann integrable on $[a,b]$. Using the decomposition of $[a,b]$ into $[a,c]$ and $[c,b]$, together with additivity of the integral, we write*

$$\int_a^b f(t)\,dt = \int_a^c f(t)\,dt + \int_c^b f(t)\,dt.$$

The decomposition supplies the structure, while additivity licenses the equality. "On the basis of this decomposition and the additivity property" would foreground the grounds of the step. A description mentioning only the decomposition may leave the mathematical justification implicit.

Example 5 (Combination with an inductive argument)**.** *Let (X,d) be a metric space, let $x^* \in X$, and let $(x_n)_{n \ge 0}$ be a sequence in X. Suppose $0 < \lambda < 1$ and $d(x_{n+1}, x^*) \le \lambda\, d(x_n, x^*)$ for every nonnegative integer n. Together with induction on n, this inequality yields*

$$d(x_n, x^*) \le \lambda^n d(x_0, x^*), \qquad n \ge 0. \tag{1}$$

The base case is equality, and multiplying the bound at step n by λ proves the next step. The phrase *together with* identifies both the recursive estimate and the inductive argument. It does not introduce an unspecified lemma or an unnumbered external inequality.

Example 6 (Explicit combination of mathematical facts)**.** *Let H be a real Hilbert space with inner product $\langle\cdot,\cdot\rangle$ and induced norm $\|h\| = \sqrt{\langle h,h\rangle}$. Let $e_1,\dots,e_N$ be an orthonormal system, where N is a positive integer, and let $f \in H$. Combining orthogonality with the nonnegativity of squared norms, we obtain Bessel's inequality*

$$\sum_{k=1}^{N} |\langle f, e_k\rangle|^2 \le \|f\|^2.$$

Indeed, for $P_N f = \sum_{k=1}^{N} \langle f, e_k\rangle e_k$, expansion gives

$$0 \le \|f - P_N f\|^2 = \|f\|^2 - \sum_{k=1}^{N} |\langle f, e_k\rangle|^2.$$

Here the two named ingredients have identifiable roles. No completeness of the orthonormal system is assumed, so an equality of Parseval type is not asserted.

Example 7 (Checking applicability)**.** *Let K be a compact metric space and let $\mathscr{F}$ be a family of continuous real-valued functions on K. Suppose $\mathscr{F}$ is uniformly bounded and equicontinuous. In view of these hypotheses, the Arzelà–Ascoli theorem applies: every sequence in $\mathscr{F}$ has a uniformly convergent subsequence with a continuous limit on K.*

Uniform boundedness means that one finite constant $M \ge 0$ bounds $|f(x)|$ for all $f \in \mathscr{F}$ and $x \in K$. Equicontinuity means that, for each $x \in K$ and $\varepsilon > 0$, there is a neighbourhood of x such that $|f(y) - f(x)| < \varepsilon$ for every $f \in \mathscr{F}$ and every $y \in K$ in that neighbourhood. The limit belongs to the space of continuous functions on K, but need not belong to $\mathscr{F}$. The phrase *in view of* draws attention to relevant hypotheses; compactness of the domain alone would not justify the application.

Example 8 (Deductive transition)**.** *In the setting of Example 5, from* (1) *and the fact that $\lambda^n \to 0$, we conclude that $x_n \to x^*$ in the metric d; that is, $d(x_n, x^*) \to 0$.*

Here *from* identifies the premises without foregrounding a particular method. It is relatively unmarked in this inferential use, but it is not devoid of meaning: it still identifies the basis of the conclusion.

6 Stylistic Enrichment and Semantic Constraints

The examples suggest that revision should begin with the function of a sentence. A writer reporting a theorem, applying a method, introducing an object, or combining estimates is performing different acts. The corresponding expressions may be related, but a change between them is justified only when it accurately describes the step being taken.

Technical terms impose stricter constraints than general rhetorical expressions. *Bounded* and *uniformly bounded* need not express the same property of a family of functions. Similarly, weak, norm, pointwise, and uniform convergence are different mathematical notions, not stylistic alternatives. A replacement that changes one of these terms may change the theorem itself.

Quantifiers require equal care. For example, if $P(x,y)$ denotes a property of elements $x \in X$ and $y \in Y$, then

$$\forall x \in X\ \exists y \in Y:\ P(x,y) \quad \text{and} \quad \exists y \in Y\ \forall x \in X:\ P(x,y)$$

have different meanings: in the first statement y may depend on x, whereas in the second one y must work for every x. Phrases such as "choose" and "fix" must respect that dependence. "There exists" asserts existence; "we may choose" also presupposes that existence has been justified in the relevant context.

Logical connectives are not freely interchangeable either. *Conversely* introduces a converse direction, whereas *on the other hand* generally signals contrast. In a conditional construction, *then* identifies the consequent and need not assert that the antecedent is true. *Therefore* ordinarily presents a conclusion from premises accepted in the current argument.

For teaching purposes, students can compare a neutral formulation with several proposed revisions, identify the relation each version expresses, and check whether the hypotheses and quantifiers remain unchanged. This is a suggested instructional activity, not an intervention evaluated in this paper. Appendix A supplies a selective guide organized by constraints rather than a thesaurus of interchangeable phrases.

7 Revision, Fixed Points, and Alternating Processes

The distinction between lexical expansion and context-sensitive refinement connects the present discussion with three related contributions. Georgiev et al. [2] propose an order-based framework for linguistic enrichment. Ilchev et al. [5] develop a local fixed point theorem for set-valued mappings in strong partial b-metric spaces. A separate paper by Ilchev et al. [6] studies alternating points of two set-valued mappings. These works address different mathematical structures; their relevance here is conceptual, and the present examples do not verify all the hypotheses of their theorems.

To describe the editing problem, fix the mathematical content, intended audience, and communicative task. Let X be a nonempty set of candidate formulations expressing the same mathematical content, although some may require grammatical or stylistic correction. For $x \in X$, let $R(x)$ be a nonempty subset of X consisting of the revisions admitted by a specified rule. A possible sequence of revisions is

$$x_{n+1} \in R(x_n), \qquad n = 0,1,2,\ldots,$$

where x_0 is the initial formulation. The rule is set-valued because several revisions may be admissible. Preservation of mathematical content is part of the construction of X and the admission rule; it is not a consequence of set-valued notation.

A fixed point x^* of this rule satisfies $x^* \in R(x^*)$. This means that retaining x^* is allowed. It does not mean that other revisions are impossible, that x^* is optimal, or that every sequence of revisions will reach it. The stronger condition $R(x^*) = \{x^*\}$ makes x^* an absorbing state of the rule, but even an absorbing state may be inadequate if the rule fails to detect a defect. For a single-valued rule $T : X \to X$, the fixed point condition is $T(x^*) = x^*$. These are statements about the chosen rule, not independent assessments of linguistic quality.

The order-based framework in [2] uses a refinement relation, denoted by $\mathbb{P}$, and a smoothing–enrichment operator. Its fixed point conclusions depend on specified order and lattice assumptions. In an editing interpretation, $(x,y) \in \mathbb{P}$ can mean that y refines x. One cannot infer that arbitrary formulations form a complete lattice merely because some revisions can be compared. Nor does

existence of an order-theoretic fixed point by itself imply that every editing sequence stabilizes after finitely many steps.

The local theorem in [5] concerns a generalized distance $p : X \times X \to [0,\infty)$ that permits positive self-distance $p(x,x)$. Its conclusions rely on conditions including zero-completeness, a suitable contraction assumption, and localization. Zero-completeness concerns the existence of a zero-self-distance limit for sequences whose pairwise generalized distances tend to zero; localization keeps the relevant iteration inside the prescribed region. These conditions cannot be inferred from the informal claim that each revision improves a sentence.

The interpretation of self-distance as linguistic deficiency requires a model-specific construction. For an elementary illustration, choose an ordinary metric d on X and a nonempty d-closed set $S \subseteq X$ of formulations accepted for the task. Define

$$\delta_S(x) = \inf_{s \in S} d(x,s), \qquad p(x,y) = d(x,y) + \max\{\delta_S(x), \delta_S(y)\}.$$

This construction is a strong partial metric and hence a strong partial b-metric with coefficient one. Moreover, $p(x,x) = \delta_S(x)$. Closedness of S ensures that $p(x,x) = 0$ if and only if $x \in S$. The construction illustrates the target-distance interpretation used in [5]; choosing a linguistically meaningful d and S remains a modelling task. No distance has been calibrated for the sentences analysed in this paper. Even in this construction, membership in S, being a fixed point of R, and being the limit of a revision sequence are distinct properties.

A simple example shows why a deficiency score alone is insufficient. Take $X = [0,\infty)$ and define the single-valued rule

$$T(x) = \begin{cases} x/2, & x > 0, \\ 1, & x = 0. \end{cases}$$

With deficiency score $e(x) = x$ and initial value $x_0 = 1$, the iterates are $x_n = 2^{-n}$. Their scores decrease to zero and the iterates converge to 0 in the usual metric, yet $T(0) = 1$, so the limit is not a fixed point. The failure here is not linguistic: it illustrates the need for hypotheses connecting the rule, the convergence notion, and the target condition.

The alternating-point framework in [6] distinguishes two successive operations. Schematically, let A and B be nonempty sets of formulations and let F assign to each $a \in A$ a nonempty subset $F(a) \subseteq B$, while G assigns to each $b \in B$ a nonempty subset $G(b) \subseteq A$. An alternating process has the form

$$b_n \in F(a_n), \qquad a_{n+1} \in G(b_n), \qquad n = 0,1,2,\ldots.$$

A compatible alternating pair $(a^*, b^*) \in A \times B$ satisfies

$$b^* \in F(a^*), \qquad a^* \in G(b^*).$$

For editing, one may interpret F as proposing a stylistic reformulation and G as checking and revising its mathematical precision. This is a proposed interpretation, not an empirically validated application. An alternating pair may contain different formulations; it need not be a common fixed point or a single terminal text. The cited paper supplies existence and convergence results under its stated assumptions, rather than a guarantee for arbitrary alternating edits.

These distinctions allow the three approaches to inform a discussion of controlled revision without replacing linguistic assessment by a formal stability claim. A concise research formulation

and a more explanatory pedagogical formulation may both be acceptable relative to their respective tasks. Whether a particular revision rule reaches either one requires a separate mathematical or empirical analysis.

8 Conclusion

The constructed examples identify conditions under which expressions associated with *from* can serve as useful alternatives in mathematical exposition. Preserving the conclusion is necessary but not sufficient: a successful reformulation must also respect the premises, the inferential relation, grammatical structure, and relevant rhetorical emphasis.

The main contribution of the analysis is therefore a set of distinctions and revision criteria, rather than an empirical claim that mathematical discourse neutralizes lexical meaning. In particular, succession should not be confused with deduction, conformity with implication, or a change in technical terminology with stylistic variation. Repetition of a precise expression is preferable when an alternative would introduce ambiguity.

The discussion of enrichment and fixed point models provides a conceptual language for admissible revisions and possible stable states. It also identifies the additional assumptions needed before stability or convergence can be asserted. Future work could test the proposed distinctions on an identified corpus of mathematical writing and evaluate reader judgements or teaching activities. Such evidence would be needed to establish how widely the illustrated preferences hold and whether the proposed guidance improves learners' writing.

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A Context-Dependent Alternatives and Their Constraints

Table 1 is a guide to possible reformulations, not a list of freely interchangeable expressions. A change may require restructuring the sentence. Technical qualifications, the direction of implication, and the scope of quantifiers must be retained.

Table 1: Selected alternatives with conditions of use

Expression	**Possible alternative**	**Constraint**
we get	we obtain; we derive; this yields	*Derive* emphasizes reasoning; *find* may describe discovery or computation.
from	by; using; applying; in view of	Match the actual source or justification. *Applying* requires an applicable result or method.
from (reporting a result)	according to	Appropriate when reporting what the cited result states; do not conceal an additional inference.
from (several premises)	combining … with …; together with	Identify all contributing premises and supply a complete main clause.
therefore	thus; hence; consequently; it follows that	Preserve the premises and conclusion; clause structure may change.
assume	suppose; under the assumption that	Preserve the scope and status of the hypothesis. *Assume further* adds a hypothesis.
denote	write … for …; introduce the notation	Naming an object and defining a new object are different operations.
there exists	there is at least one	*We may choose* is appropriate only after existence is justified; preserve dependence on earlier parameters.
for every	for each; for all; given an arbitrary	Retain the domain and quantifier scope. *For a fixed* requires an arbitrariness argument if a universal conclusion is intended.
if	provided that; under the condition that	Preserve the direction of implication and any implicit universal quantification.
then (in if–then)	an explicitly stated consequent	*Therefore* is not an automatic replacement inside a conditional construction.
we have; there holds	the following relation holds	*Identity*, *estimate*, and *inequality* describe different kinds of relations.
let us consider; consider	we now consider; examine; study	*Take into account* indicates use as a factor or premise, not merely selection of an object of study.
it follows	we conclude; we deduce; this implies	Name or make clear the premises. *This proves* asserts completion of a specified claim.

Table 1 (continued)

Expression	**Possible alternative**	**Constraint**
because	since; as; because of	*Because of* takes a noun phrase; *because* can introduce a clause. *Since* and *as* may be ambiguous.
clearly	a short justification; directly from …	*Obviously* does not supply a missing proof and can be unhelpful to learners.
now	next; at this point	*Moreover* adds information; it does not simply mark the next step.
similarly	by the same argument; analogously	Identify any changed hypotheses or steps. Similarity is not a proof when the earlier argument no longer applies.
finally	at the final step; to complete the proof	*In conclusion* may summarize; *it remains to* announces unfinished work.
we prove; show	we establish; we demonstrate	*Suggest*, *indicate*, and *confirm numerically* do not assert a proof.
choose	select; fix; take	Existence must be known; *fix* keeps the selected object constant within the relevant argument.
satisfies	has the property that; meets the condition	Preserve the exact condition. *Is consistent with* can express a weaker relation.
bounded	bounded in the specified space	*Uniformly bounded*, *bounded above*, and *bounded below* are distinct qualifications, not automatic synonyms.
converges	tends to; has limit	Preserve the mode and topology of convergence. Weak, norm, pointwise, and uniform convergence are not interchangeable.
the proof is complete	this completes the proof; the claim follows	All cases and required directions must already have been established.
using the inequality	by the inequality; applying the inequality	The inequality's hypotheses and the values substituted into it must be valid.
we get the inequalities	we obtain the inequalities; we derive the estimates	Use *chain of inequalities* only when a chain is actually displayed.
on the other hand	however; by contrast	*Conversely* introduces a converse direction and is not a general contrast marker.
in particular	as a special case	*More precisely* refines a statement; *notably* emphasizes it. Neither necessarily introduces a special case.
it is enough to show	it suffices to prove	The proposed claim must imply the required conclusion. *It remains to* also refers to the stage of the proof.
under these assumptions	under the stated hypotheses; under these conditions	Retain every assumption needed for the step; a vague reference to a setting may be insufficient.
by definition	from the definition; directly from the definition	The conclusion must actually follow from the cited definition.

Table 1 (continued)

Expression	Possible alternative	Constraint
we conclude that	we deduce that; it follows that	Preserve the conclusion and its logical dependence on the preceding argument.
provided that	if; under the condition that	Do not replace a sufficient condition by a necessary condition or introduce an equivalence.

Vanya Ivanova
Affiliation: Department of Computer Systems, Faculty of Mathematics and Informatics, University of Plovdiv Paisii Hilendarski, 4000 Plovdiv, Bulgaria
ORCID ID: 0000-0002-0043-8606
E-mail: vantod@uni-plovdiv.bg

Angel Todorov
Affiliation: Faculty of Philology, University of Plovdiv Paisii Hilendarski, 4000 Plovdiv, Bulgaria
ORCID ID: 0009-0008-3704-2761
E-mail: angel.todorov@uni-plovdiv.bg

Boyan Zlatanov
Affiliation: Department of Mathematical Analysis, Faculty of Mathematics and Informatics, University of Plovdiv Paisii Hilendarski, 4000 Plovdiv, Bulgaria
ORCID ID: 0000-0002-5857-9372
E-mail: bobbyz@uni-plovdiv.bg